

7.5 Roots and Zeros

Fundamental Theorem of Algebra

- all polynomial equations with a degree greater than zero have at least one root in the set of complex numbers
- a polynomial equation of the form $P(x) = 0$ of degree n with complex coefficients, has exactly n roots

How many roots?

ex:

$$a - 10 = 0$$

1

$$x^2 + 2x - 48 = 0$$

2

$$3a^3 + 18a = 0$$

3

$$y^5 - y = 0$$

5

Solve # 3.

$$3a^3 + 18a = 0$$

$$3a(a^2 + 6) = 0$$

$$a = 0 \quad a^2 + 6 = 0$$

$$a^2 = -6$$

$$a = \pm i\sqrt{6}$$

1 real 2 imaginary

Solve # 4.

$$y^5 - y = 0$$

$$y(y^4 - 1) = 0$$

$$y(y^2 + 1)(y^2 - 1) = 0$$

$$y = 0 \quad y = \pm i \quad y = \pm 1$$

3 IR
2 imag.

Complex Conjugates

 $a, b \in \mathbb{R} \quad b \neq 0$ If $a + bi$ is a zero, then $a - bi$ is a zero1. If $3 + 2i$ is a zero, then $3 - 2i$ is also a zero.2. If $5 - 3i$ is a zero, then $5 + 3i$ is also a zero.3. If $1 - \sqrt{5}$ is a zero, then $1 + \sqrt{5}$ is also a zero.
(It also works for radicals)4. If $6i$ is a zero, then $-6i$ is also a zero.

Write a function with the following zeros.

3, $2 + i$, $2 - i$ $\text{sum} = 4$
 $\text{prod} = 5$

$$(x-3) \quad x^2 - \text{sum}x + \text{product} = 0$$

$$(x-3)(x^2 - 4x + 5) = 0$$

$$x^3 - 7x^2 + 17x - 15 = 0$$

Write a function with the following zeros.

 $-2, 2 - \sqrt{3}$, $2 + \sqrt{3}$

$$\text{sum} = 4 \quad (x+2)(x^2 - 4x + 1)$$

$$\text{prod} = 1 \quad \begin{array}{l} x^3 - 4x^2 + x \\ + 2x^2 - 8x + 2 \\ \hline x^3 - 2x^2 - 7x + 2 = 0 \end{array}$$

Write a function with the following zeros.

-8, $5i$

HW

p375-376

13, 15, 17,

35, 38, 39, 41